Bulgarian Society for NDT
International Journal "NDT Days"
Volume VIII, Issue 4, Year 2025
ISSN: 2603-4018
eISSN: 2603-4646

Development of a Simplified Probabilistic Approach to Assessing the Cyclic Strength of Cylinders under Conditions of Limited Technical Information and a Small Number of Loads. Scale Factor.

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Abstract
The analysis of regulatory documentation on cyclic tests of gas cylinders and literature sources on cyclic strength and durability dispersion depending on the stress level is conducted. Normal and log–normal distributions describing durability dispersion are considered. A model of probabilistic assessment of possible cylinder failure under cyclic loads depending on the number of cylinders tested by a predetermined number of cycles was constructed. The cylinders tested should not "destroy" – lose their sealing. Dependencies for taking into account the scale factor are given.

Keywords: durability, cylinders, cyclic testing, pressure, dispersion, median, scale factor, safety factors, lognormal distribution.

1. General terms

When approving a new type of high–pressure gas cylinder, in order to prove its operational reliability, along with a fairly wide range of tests, cyclic tests with internal hydraulic pressure are also carried out. In most cases, these tests are carried out once, at the very beginning. But according to some regulatory documents (RD), in order to confirm the stability of manufacturing quality, cyclic tests are also carried out during the established production of cylinders, for example, one of a batch, or one of five or ten consecutive batches. Detailed test procedures can be found in the relevant regulatory documents for the manufacture of certain cylinders. Some provisions of cyclic tests are common for a number of RD.
Cylinders are divided by design type: CNG–1 – metal; CNG–2 – metal liner reinforced with wire or continuous fibers (ring winding); CNG–3 – metal liner reinforced with continuous fibers (full winding, also called cocoon winding); CNG–4 – continuous fibers, with a non–metallic liner (fully composite). CNG–3 type cylinders are divided into two subtypes: in one, the liner takes on part of the design load (more than 5%), in the other, it serves only for sealing. The requirements for cyclic testing of fully composite cylinders (CNG–4), in which the only metal parts are embedded elements, are essentially the same as for other cylinders. However, unlike the others, they are checked for gas permeability in the process of cyclic testing, after a certain

number of cycles. Some of the cycles for such cylinders, as well as for CNG–3 type cylinders with a non–load–sharing liner, are carried out under vacuum.
When approving a new type of cylinder, 2–3 cylinders are subjected to cyclic tests. These tests are usually carried out with a test hydraulic pressure, which in most cases is 1.5 times the working pressure. Cylinders with an unlimited service life must withstand 12,000 cycles of test pressure, or according to some regulatory documents 24,000 cycles of a pressure, that could be achieved in the cylinder (taking into account the properties of the gases used) at a temperature of 65 ^{o}C. The tests must be passed without unsealing (failure due to cracks or leakage due to a fistula). In the case of high proof pressure, tests may be carried out with a pressure equal to 2/3 of the proof pressure. In this case, the cylinders must withstand 80,000 cycles without unsealing. For cylinders with a limited service life, the tests consist of two consecutive parts. In the second part, unsealing of the cylinder is allowed due to the formation of a fistula, which occurs due to a fatigue crack, and not a fracture due to quasi–static failure. In some cases, additional cyclic tests are carried out for several cylinders with artificial defects, but with a pressure lower than the proof pressure and a smaller number of cycles. According to some regulatory documents, a fistula may form after a certain period of operation. Also, cyclic tests are carried out on cylinders after drop tests onto a plate, after hitting a sharp edge (and at negative temperatures), and, if necessary, on cylinders after tests in salt water. Additional cyclic pressure tests are also carried out at extreme temperatures and after short–term exposure to an open flame. Sometimes, after cyclic tests, including additional ones, cylinders are brought to destruction by static pressure with determination of safety factors. Since high–pressure cylinders are objects of increased danger, the range of their various tests is quite wide. More detailed information on the requirements for cyclic tests for various regulatory documents is given in [1].
High–pressure gas cylinders are subject to cyclic testing. Low–pressure cylinders, such as welded steel cylinders for 1.6 MPa for liquefied hydrocarbon gases, etc., are not subject to cyclic load testing. It should be noted that cyclic tests with internal pressure are also not carried out in the pipeline industry, since the wall thickness safety factor is deliberately high there.
The working pressure in the cylinder is set by the gas pressure at a temperature of 15 ^{o}C. According to some outdated RD 20 ^{o}C. At the maximum operating temperature of the cylinder, the pressure in it should not exceed the proof pressure. As we can see, the working pressure of the cylinder is very conditional. The proof pressure is the designed pressure of the cylinder. The stresses in the walls at such a pressure should not exceed 90% of the flow stress. At the same time, the proof pressure for cylinders made of a material whose ratio of temporary resistance to the flow stress is more than 2 can be reduced to 1.25 of the working pressure. For cylinders made in the USA, the coefficient 5/3 is common.
In a number of RD, cyclic tests are carried out with test pressure. And, obviously, in the case of non–destruction, a greater guarantee of cyclic durability and reliability is obtained in comparison with those RD, where tests are carried out at the level of working pressure. It is also important that at higher loads, the probability of detecting quasi–static destruction inherent in soft loading increases – destruction due to opening (fracture), which is unacceptable for cylinders. And such destruction can occur in the case of imperfections in design, for example, shapes of concave bottoms, places of transition from one geometric shape to another, etc. Local defects, inclusions, cavities, cracks, etc. that operate under conditions of hard loading can also lead to unsealing, but it mainly occurs within the fatigue mechanism due to the formation of a fistula, which is less dangerous compared to a rupture.
Fatigue durability (fatigue life) of the structure N_f – the total number of cycles from the beginning of the test to failure, at stresses above the fatigue limit. It consists of the number of cycles before the formation of a crack of a predetermined size (durability to crack) and survivability (residual durability). There is an opinion that when testing metals, the shape of the loading curve does not affect fatigue durability. Fatigue failure is determined only by the

highest and lowest stresses [2]. Experiments also show, that effect of the frequency of stress change is insignificant. Exceptions are tests at high temperatures, as well as under the influence of a corrosive environment. Under these conditions, a decrease in frequency leads to some decrease in fatigue resistance [2]. Residual stresses act similarly to average cycle stresses [3]. With increasing stresses, N_f decreases.

In fatigue testing, there are two significantly different types of loading: loading with a given load range – soft loading (movements are not kinematically limited); loading with a given deformation range – hard loading. Most common and most consistent with the mass service conditions of structural parts in operation are tests with a given load range. With hard loading, there is no accumulation of deformation, which eliminates the possibility of quasi–static failure. In this case, all materials fail according to the fatigue type with the formation of cracks [4].

In no other type of failure do strength characteristics depend on such a large number of factors as in fatigue failure. The main ones are: characteristics of the material and manufacturing technology; design of the product/part; loading conditions; environment in contact with the product/part [5]. In low–cycle loading, the determining factor is the relationship between complexes of basic mechanical properties and durability, rather than the relationship between individual basic mechanical properties and durability [6]. The basis for probabilistic approaches is the fact that the processes of accumulation of fatigue (cyclic) damage, leading to the initiation and subsequent development of fatigue (including low–cycle) cracks, are random in nature [6]. Under the same stress, the tested metal samples can show the number of cycles before failure that differ by one or even two orders of magnitude [3]. In the works of many authors, a considerable spread of cyclic deformation characteristics is observed, in many cases significantly exceeding the spread of the basic mechanical properties. The statistical nature of the fatigue failure process predetermines the dispersion of fatigue test results to a greater extent than other types of tests [5]. The coefficients of variation for survivability are not less than 0.2 – 0.5 [6]. According to reference data, during durability tests, the coefficient of variation reaches values of 0.2 – 0.3. Data dispersion is also observed when determining conventional strength limits, flow strength and other characteristics, but with a relatively low coefficient of variation of 0.01 – 0.05.

In fatigue tests, the dispersion of durability decreases with increasing stress level [5, 6, 7, 8]. For this reason, the dispersion of durability decreases with increasing stress concentration level [5]. With increasing stress level, the coefficient of durability variation also decreases [6, 9, 10]. With increasing level of acting stress, a transition from grain–wise fracture to grain boundaries fracture is observed, location and nature of fracture may change [5]. In field tests, the amplitude range at the most dangerous points is very difficult to determine reliably, in contrast to the number of cycles at which the fracture occurred.

The dispersion largely depends on the shape and size of the samples. With an increase of across dimensions of tested metal samples, the dispersion in durability decreases [3, 5]. It has been established experimentally that small metal samples are stronger than large ones. In samples of large cross–sections, compared to small ones, there is a greater probability of the presence of defects and dangerously stressed grains, which is associated with the statistical nature of the fatigue failure process [3].

The standard safety factor for durability is equal $\bar{n}_N = \frac{N_{f1/2}}{[N]}$ to [11], where $N_{f1/2}$ is the number of cycles to failure with a probability of 0.5 (50%), is the median of the fatigue life distribution, $[N]$ is the maximum acceptable number of cycles at which the probability of failure is extremely low, practically zero. In practice, technical specifications (TS) is usually assume $\bar{n}_N = 10$, for certainty. Such factor value is applied to pressure vessels, pipelines, and other products with a low initial stress concentration. For structural elements and machine parts with a deliberately high initial stress concentration, due to lower dispersion, $\overline{n}_N = 3.0$ is used [12]. When calculating aircraft engine components, for example, $\overline{n}_N = 5 \ldots 10$. If full–scale structures or full–scale models are subjected to testing under operational loads, then according to [12] $\overline{n}_N \geq 2.1$ is used.

Taking into account the literature sources regarding the distribution of fatigue (cyclic) durability of experimental samples, most authors tend to the log–normal distribution law. In this case, the variation coefficient characterizing the dispersion can fluctuate within the region of 0.2...0.5. Using this distribution and the variation coefficient, for example 0.5 for cylinders (the worst one), it is possible to predict their cyclic durability based on the results of preliminary cyclic tests without destruction. The probability distribution of possible destruction depends on the number of cylinders on which the tests were conducted and on the number of loading cycles during these tests. The tested cylinders should not lose their tightness "fail" during the tests.

The requirements specified in RD apply to cylinders with an uncontrolled, large number of cyclic loads during operation, which in turn can be unlimited. If the number of cyclic loads of cylinders during their operation is small and controlled, then RD requirements are too high. Therefore, for such cylinders, it is recommended to conduct cyclic tests with a smaller number of cycles, but this must be justified. This technique can be used as an express method, for example, in the case of an increase in test/working pressure. If this increase is justified by static strength. Or for cylinders that are not manufactured in compliance with any RD. This technique is based on cyclic tests with a small number of cycles that do not lead to failure. Of course, there are many methods for calculating cyclic strength, but this requires knowledge about geometry, mechanical properties, possible imperfections in geometry leading to stress concentration, etc. This paper does not consider methods for calculating cyclic strength.

A detailed elaboration of the theory presented below is considered in [1]. All mathematical calculations, which are given in this paper without detailed proofs, are also given there. The dependencies given in the article for normal and log–normal distributions, as well as these distributions themselves, were verified using Monte Carlo method. The same applies to the theoretical distributions of their minimum values.

2. Application of normal and log–normal laws to estimate the distribution of durability

Let us consider a discrete random variable X, which is defined by its finite sample $x_1, x_2, \ldots, x_i, \ldots, x_n$ of n elements. Regardless of the law of distribution of this random variable, the sample mathematical expectation (sample mean) M_x and sample mean square (standard) deviation σ_x as the square root of the unbiased estimate of the sample variance are valid for it.

$$M_x = \frac{1}{n} \sum_{i=1}^{n} x_i \,, \; \sigma_x = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} \left(x_i - M_x \right)^2} \,. \tag{1}$$

The distribution function of a discrete variable X: $F(x) = P(X < x)$, where $P(X < x)$ is the probability that the variable X takes a value less than a predetermined specific number x. For any random variable, discrete or continuous, the following is true: $0 \leq F(x) \leq 1$. The more elements in the sample, the more "reliably" the given characteristics determine the value X. If the value X consists of an infinite number of elements, then when $n \Rightarrow \infty$ the sample tends to the general population. But there may be exceptions when the number of elements of the variable X is finite, then this number will be the general population.

Given values of M_x, σ_x and the distribution law of a discrete random variable, it can be replaced by a continuous random variable with a larger functionality. A useful function, inherent only to continuous random variables, is the probability distribution density: $f(x)=\frac{dF(x)}{dx}$, where $F(x)=P(X<x)=\int_a^x f(x)dx$ is the distribution function for a continuous variable. The lower value of the integration interval a is the lower boundary of the region of acceptable values of X. In case of a normal distribution $a=-\infty$, for a log–normal distribution (see below), this boundary is zero. The area under the curve $f(x)$ is naturally equal to one. For discrete random variables, there is no probability distribution density function, since such random variables are not absolutely continuous functions. If $F(x)$ for a discrete variable is a step function, then for a continuous variable it is smooth. The mathematical expectation and variance (mean square deviation squared) of a continuous variable are determined by the following dependencies: $M_x=\int_a^\infty xf(x)dx$, $\sigma_x^2=\int_a^\infty (x-M_x)^2 f(x)dx$. Further, we will not specify whether this is a discrete or continuous variable, but will use the general mathematical apparatus.

Additional equally important numerical characteristics of a random variable X.

Me_x – median – the value of a random variable X for which the value of the distribution function is 0.5 – the value dividing areas with 50 % probability. The number of sample elements, for a discrete variable, to the right and left of the median is approximately the same.

Mo_x – mode – the value of a random variable X with the highest frequency – the value at which the maximum of the probability distribution density function is observed – $f(x)$.

$v_x=\sigma_x/M_x$ – coefficient of variation of a random variable X (relative standard deviation).

A conditional classification of samples based on the coefficient of variation is used: when $v_x \le 0.1$ the sample is weakly variable, when $0.1 \le v_x \le 0.2$ it is moderately variable, and when $v_x \ge 0.2$ it is highly variable.

For a normal distribution, the mathematical expectation, median, and mode coincide. It is worth noting that if we consider a sample, then the values of the median, mode, and variation coefficient will be sample values. It is clear that the more elements in the sample, or the closer the sample to the general population, the more "reliable" these values are. Based on the sample numerical characteristics obtained during the processing of test results, using additional mathematical apparatus, conclusions about the law of distribution of a random variable are made. Or which distribution low a random variable obtained by experimental methods most closely corresponds to. This requires large samples. After we have proven or guessed which distribution law the random variable corresponds to, we can use its mathematical apparatus to solve specific problems.

If the random variable X is distributed according to the normal law (Gaussian distribution), then its distribution density and distribution function are respectively equal to:

$f(x)=\frac{e^{-t^2}}{\sigma_x\sqrt{2\pi}}$, $F(x)=\frac{1}{2}(1\mp erf(t))$, where the minus is taken for values located to the left of the mathematical expectation, $t=k/\sqrt{2}$ is a positive value, $k=|x-M_x|/\sigma_x$ are deviations from the mathematical expectation in standard deviations, and $erf(t)$ is the Gaussian error function.

To construct the density and distribution function of a value X distributed according to the normal law, as well as to perform various calculations, it is recommended to represent the x–axis as $x=M_x\pm k\sigma_x$, ($k>0$). In this representation, the numerical values on the x–axis are symmetrical with respect to the mathematical expectation.

Parameter $k=(x_i-M_x)/\sigma_x$ shows how many standard deviations the value x_i is from the mathematical expectation M_x to the right or left, depending on the sign. In case of normal distribution $F(x)\equiv F(k)\equiv P(X<x)$. Value k can be represented as the result of the inverse normal distribution function $k=F^{-1}(P(X<x))$. Values k and $F(k)$ are given in mathematical statistics reference tables.

A log–normal distribution is a distribution of a strictly positive quantity W, ($W>0$), whose logarithm $U=ln(W)$ is normally distributed [13]. The values of U can be both greater than and less than zero. Element–wise, it can be represented as $u=ln(w)$, or $w=e^u$, where w is an element of set W, and u is an element of set U. The distribution density and the distribution function of the quantity U are naturally equal to the density and the normal distribution function. The log–normal distribution is more convenient to use than the normal distribution.

In case of continuous representation, from the relationship of quantities U and W, and since $\frac{dw}{du}=e^u=w$, we have $f(w)=\frac{dF(w)}{dw}=\frac{dF(w_u)}{du}\frac{du}{dw}=f(u)\frac{du}{dw}=f(u)\frac{1}{w}$. Substituting what was found for the distribution density into the distribution function of quantity W we obtain $F(w)=\int_0^w f(w)dw=\int_0^w f(u)\frac{dw}{w}=\int_{-\infty}^u f(u)du=F(u)$. From this, it can be seen that distribution functions of quantities U and W coincide.

We will take the mathematical expectation and variance of a random variable (W), that follows a log–normal distribution, from the textbook [13]:

$M_w=e^{M_u+\frac{\sigma_u^2}{2}}$, $\sigma_w^2=e^{2M_u+\sigma_u^2}\times\left(e^{\sigma_u^2}-1\right)$, where M_u and σ_u^2 are mathematical expectation and variance of quantity U. Dividing σ_w^2 by M_w^2, we obtain $\frac{\sigma_w^2}{M_w^2}=\left\{v_w^2\right\}=e^{\sigma_u^2}-1$, hence:

$$\sigma_u=\sqrt{ln\left(\frac{\sigma_w^2}{M_w^2}+1\right)}=\sqrt{ln\left(\nu_w^2+1\right)}, \tag{3}$$

where $\nu_w=\sigma_w/M_w$ is the coefficient of variation of the quantity W.

From the expression for M_w we find

$$M_u=ln\left(M_w\right)-\frac{\sigma_u^2}{2}=ln\left(M_w\right)-\frac{1}{2}ln\left(\nu_w^2+1\right)=ln\left(M_w\Big/\sqrt{\nu_w^2+1}\right).$$

It is clear that $M_u=M_{ln(w)}\neq ln\left(M_w\right)$ and $\sigma_u=\sigma_{ln(w)}\neq ln\left(\sigma_w\right)$.

Dependencies, obtained for a continuous variable, also work for a limited sampling distribution, provided that it is actually distributed according to a lognormal law. We need quantity U to use its distribution function as for a normal law, nothing more.

Distribution density and distribution function of a log–normally distributed variable W are equal respectively:

$$f\left(w\right)=\frac{1}{w\sigma_u\sqrt{2\pi}}\cdot e^{-\frac{\left(ln(w)-M_u\right)^2}{2\sigma_u^2}}=\frac{e^{-t^2}}{w\sigma_u\sqrt{2\pi}},$$

$F\left(w\right)=\frac{1}{2}\mp\frac{1}{2}erf\left(\frac{\left|ln(w)-M_u\right|}{\sigma_u\sqrt{2}}\right)=\frac{1}{2}\left(1\mp erf\left(t\right)\right)=F\left(u\right)$, where the minus sign is taken for values to the left of the mathematical expectation of the logarithm, parameter t (see above).

The median and mode of the lognormal distribution are defined by the following expressions: $Me_w=e^{M_u}=M_w\Big/\sqrt{\frac{\sigma_w^2}{M_w^2}+1}=M_w\Big/\sqrt{\nu_w^2+1}$, $\quad Mo_w=e^{M_u-\sigma_u^2}=M_w\Big/\sqrt{\left(\nu_w^2+1\right)^3}$. The condition is met: $Mo_w < Me_w < M_w$. It is useful to point out that mathematical expectation of the logarithm is directly converted into the median of the lognormal distribution.

The coefficient of variation of the variable $U=ln\left(W\right)$, can be expressed in the following way:

$$\nu_u=\frac{\sigma_u}{M_u}=\frac{\sqrt{ln\left(\nu_w^2+1\right)}}{ln\left(M_w\Big/\sqrt{\nu_w^2+1}\right)}=\frac{\sqrt{ln\left(\nu_w^2+1\right)}}{ln\left(Me_w\right)}.$$

With the variation coefficient ν_w and therefore ν_u tending to zero, the lognormal distribution tends to normal, therefore median and mode approaches to mathematical expectation. It is worth noting that the greater the variation coefficient, the more Mo_w, Me_w and M_w differ from each other. And also $\nu_u << \nu_w$, which gives advantages to the log–normal law.

If a random variable W is really distributed according to a lognormal law, then it is sufficient to operate with the values M_w and σ_w to determine all dependencies for it. These values can be found, according to the results of cyclic tests, using formulas (1). If we assume a normal distribution law, then we also use formulas (1).
To construct the density and distribution function of a variable W distributed according to a lognormal law, as well as to perform various calculations, the x–axis can be taken as for the variable X (see above), but it is more appropriate to represent it in the form $w=e^u$, where $u=M_u \pm k\sigma_u$. After substituting, we have,

$$w=e^u=e^{M_u \pm k\sigma_u}=e^{M_u}\cdot e^{\pm k\sigma_u}=Me_w\cdot e^{\pm k\cdot\sqrt{ln\left(v_w^2+1\right)}}=Me_w\left(e^{\sqrt{ln\left(v_w^2+1\right)}}\right)^{\pm k}.$$

In this representation, numerical values for variable U on the x–axis are symmetrical with respect to median.

As an illustration, Figure 1 shows the normal and lognormal distributions.

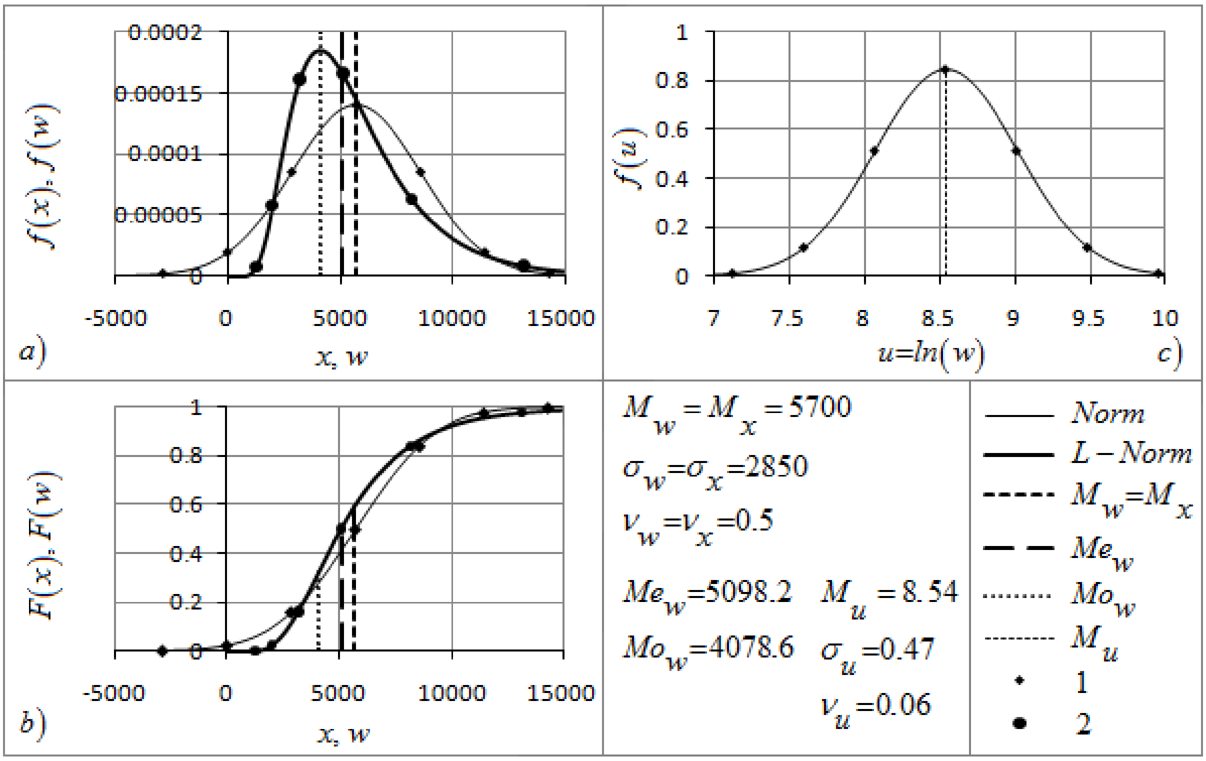

Fig. 1. Normal and log–normal distributions with a mathematical expectation of 5700 and a variation coefficient of 0.5:

Norm – normal distribution; L–Norm – lognormal distribution; 1 – for $x=M_x \pm k\sigma_x$, 2 – for $w=e^{M_u}\cdot e^{\pm k\sigma_u}$, values for the x–axis with deviations multiples of the standard deviation, where $k=-3, -2, -1, 0, 1, 2, 3$.

The relationship between k_u and k_w depends only on the coefficient of variation.

Taking $k_u=\frac{M_u-u_i}{\sigma_u}$, it can be shown that in case of representation $k_w=\frac{M_w-w_i}{\sigma_w}$, it follows that $\frac{M_w}{w_i}=\frac{1}{1-k_w\nu_w}$ and $k_u=\frac{-ln\left(\left(1-k_w\nu_w\right)\sqrt{\nu_w^2+1}\right)}{\sqrt{ln\left(\nu_w^2+1\right)}}$.

If we assume that $k_w=\frac{Me_w-w_i}{\sigma_w}$, then $\frac{M_w}{w_i}=\frac{1}{\frac{1}{\sqrt{\nu_w^2+1}}-k_w\nu_w}$ also $k_u=\frac{-ln\left(1-k_w\nu_w\sqrt{\nu_w^2+1}\right)}{\sqrt{ln\left(\nu_w^2+1\right)}}$.

3. Assessment of probability cylinder failure based on the results of testing their prototypes without failure

The million cylinder problem. Let's hypothetically assume that we have one million cylinders, each of which will be filled 10 times with the working pressure during operation – a small number of loading cycles. We need to conduct an experimental express test with some guarantee that all our cylinders will withstand the operational number of loads (10 times). We know nothing about the cylinders: material, maximum pressures, geometry, mechanical properties, stress concentration factors in most dangerous places, distribution of cyclic (fatigue) durability, literally, nothing. We only know that cylinders belong to the same type, and even better, to the same production batch. Cylinders have passed non–destructive testing and preliminary loading with a proof pressure that exceeds the operating pressure by some amount. Cyclic loading of randomly selected test cylinders will be carried out with a proof pressure. During the tests, the cylinders should not "fail". First, we load one cylinder with 10 pressure cycles. The result does not say much. If we give it a fivefold load, i.e. 50 cycles, and it does not fail, then there is some insignificant probability that our million cylinders will withstand 10 cycles. If we load 2 cylinders with 50 cycles, and they do not fail, then the probability will be slightly higher than with only one cylinder. Let's imagine our thought experiment as a series of elements (number of cylinders – number of loading cycles): 1 – 10; 1 – 50; 2 – 50; 2 – 100; 3 – 100; 4 – 100, etc. It is intuitively clear that the more cylinders involved in the tests, and the greater the number of their loadings with test pressure, the greater probability that our million cylinders will withstand 10 cycles with the working pressure.

It is also intuitively clear that to increase this probability, one must either increase the number of cylinders in the tests or increase the number of their loading cycles. The question is how to estimate these probabilities.

The issue of choosing cylinders for cyclic testing is also important. Random selection is not optimal. If we do conduct cyclic testing, it is better to choose the worst cylinders of all approved. Such cylinders can be those that have:

- a higher coefficient of residual expansion under test pressure loading after production, or
- greater elastic expansion (if the residual is unknown), or
- a higher volume–to–weight ratio (if the residual and elastic expansions are unknown), or
- there are some different, but acceptable defects.

In accordance with some regulatory documents, elastic and residual expansion are estimated for cylinders. According to these criteria, cylinders are rejected both at the production stage and during their periodic inspection. These criteria, in those RDs where they are provided, are applied to each cylinder when it is loaded with test pressure, i.e. 100 % control [1, 14, 15, 16]. Probability that all out of n the randomly selected numbers (elements) will be greater or less than median is $0{,}5^n$. As n increases, this probability decreases. For $n=[1, 2, 3, 4, 5]$, these probabilities will be respectively: 0.5; 0.25; 0.125; 0.0625; 0.03125. For example, we tested 3 cylinders with a certain number of cycles, and they did not fail. Probability that this number of cycles will be greater than median is 12.5 %, for 5 cylinders, respectively, 3.125 %. In case of testing only one cylinder, it is clear – 50%.

Since $F(x)=P(X<x)$, where x is a given value, the probability that n random numbers are simultaneously less than x is equal to the product of these probabilities, i.e.: $(F(x))^n$. The probability that these n numbers are simultaneously greater than x is equal to $(1-F(x))^n$. For simplicity, the function $F(x)$ can be replaced by $F(k)$, where $k=(x-M_x)/\sigma_x$. For example, x can be the number of cycles to failure in case of a normal distribution or natural logarithm of the number of cycles to failure in case of a log–normal distribution. M_x and σ_x are mathematical expectation and standard deviation of the number of cycles to failure or the logarithm of the number of cycles to failure. Functions $(F(k))^n$ and $(1-F(k))^n$ in case of a normal distribution, for $n=[1, 2, 3, \quad 200]$ are shown in Figure 2, they are symmetrical to each other.

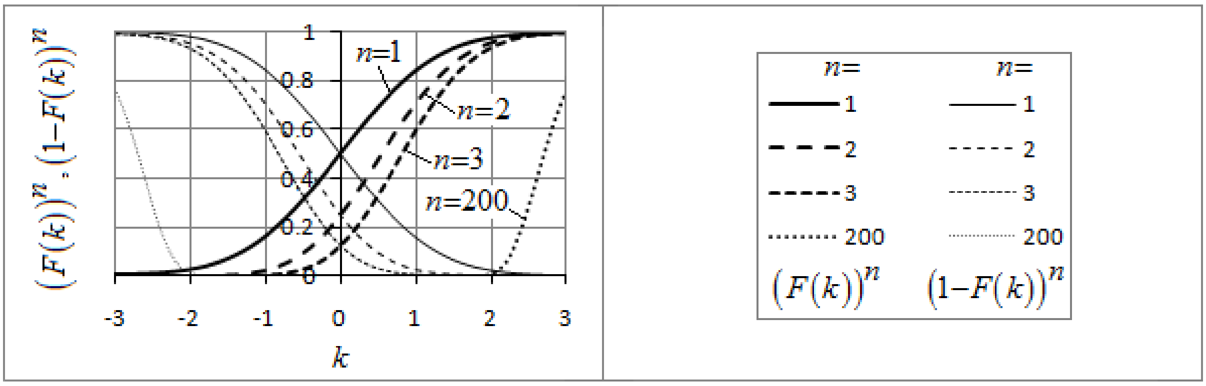

Fig. 2. Functions $(F(k))^n$ and $(1-F(k))^n$ in case of normal distribution, for $n=[1, 2, 3, \quad 200]$.

In case of testing one sample ($n=1$), it is clear that function $(F(k))^n$ coincides with the distribution function $F(k)$, and $(1-F(k))^n$ is equal to $1-F(k)$. These functions sum up to one, for $n=1$.

Suppose we tested $n_c=3$ cylinders with N_t number of test loading cycles without failure, and none of the cylinders "failed". Further, such a case, in which we have not received failure during cyclic testing, be it a fracture or a fistula leading to loss of tightness, will be called a "successful" test. Another criterion can also be chosen as a successful test. For composite cylinders, this can be, for example, non–critical failure of fibers, non–peeling of the outer protective coating, etc. In this case, we know nothing about the real distribution of the number of cycles before failure.

If we assume that with our N_t cycles we are in the median zone of the real distribution, then this probability is equal to $\left(1-F\left(N_t\right)\right)^3=(1-0.5)^3=0.5^3=0.125$, see Figure 2, curve 3 – a thin line. The probability that all three of them should already fail is also equal to 0.125, curve 3 – a thick line. If we assume that by some reason we ended up, for example, in the zone of mathematical expectation plus 1 standard deviation, without receiving a single destruction, then such a probability will generally be very low $\left(1-F\left(N_t\right)\right)^3=(1-0.841345)^3=0.158655^3=0.003994$. Moreover, the probability that all three of them should already be destroyed is equal to $\left(F\left(N_t\right)\right)^3=0.841345^3=0.595555$.

Let's assume that we have tested n_c cylinders with a test $N_t>[N]$ number of loading cycles without failure, a "successful" test. If, when loading n_c+1 cylinder, it fails after a number of cycles equal to or less than N_t, then failing probability will be $1/\left(n_c+1\right)$. As n_c increases, probability decreases. This number can be equated to the probability $P_{(-)}$ that this cylinder will not withstand N_t the cycles, provided that all the previous ones have passed the tests, i.e.

$$P_{(-)}=F\left(N_t\right)=1/\left(n_c+1\right). \tag{4}$$

Also, probability $P_{(+)}$ that n_c+1 cylinder will withstand N_t cycles is $P_{(+)}=1-F\left(N_t\right)=n_c/\left(n_c+1\right)$. These two probabilities sum up to one. It is worth noting that if $n_c=1$ (one cylinder was tested), then the probability that the next cylinder will withstand the tests is 50 %. The values of the probability of failure and non–failure, depending on the number of cylinders tested, are shown in Figure 3a. Figure 3b shows the value of parameter $|k|$ (it is clear that k is negative), as a result of inverse normal distribution function from the values $P_{(-)}$, $P_{(+)}$, which are mutually symmetric.

It should be noted that instead of "cylinders" can also be some other full–scale samples or elements.

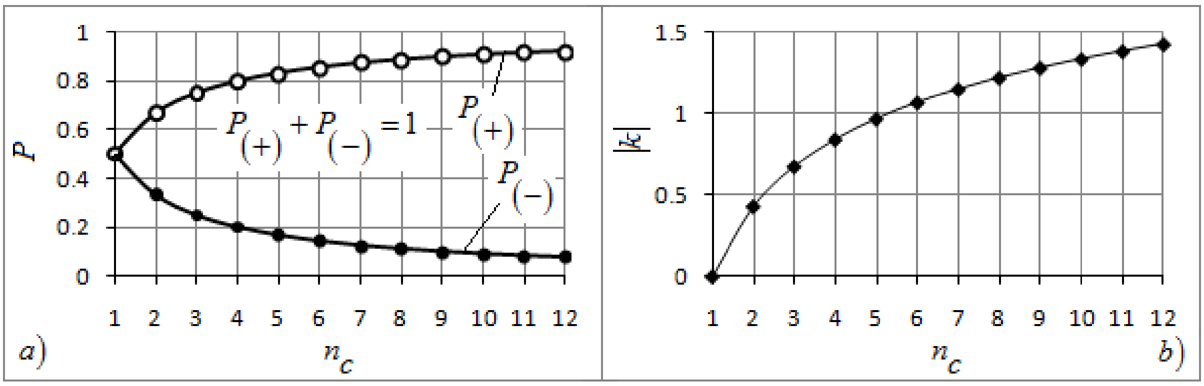

Fig. 3. Probabilities of failure and non–failure and values of the inverse normal distribution function depending on the number of cylinders tested:

a – probabilities of failure $P_{(-)}$ and non–failure $P_{(+)}$; b – value of parameter $|k|$, as a result of inverse normal distribution function.

Without actual values N_f, one can guess that with an increase of the number of "successfully" tested cylinders n_c, the conditional median $N_{f1/2}$, for which the probability of failure is 50%, shifts to the right w.r.t. N_t, see Figure 4. (This shift will be greater for larger variation coefficients). When testing only one cylinder, N_t and $N_{f1/2}$ coincide. The dots in Figure 4 indicate the values for testing 1, 2, 3, 4, 5, 10, 20, 30, 40, 50, 100 cylinders (from right to left).

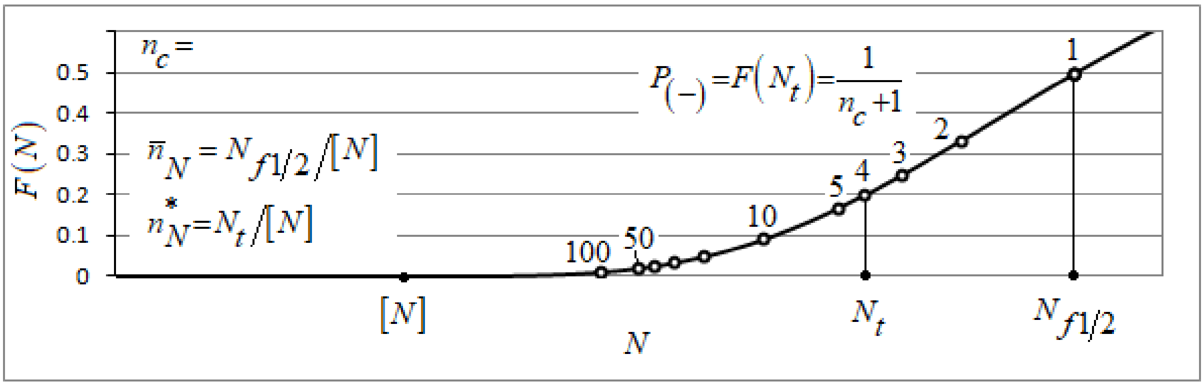

Fig. 4. On the idea of cyclic testing of cylinders without failure.

Having the values of N_t and n_c, using formula (4), we obtain the probability of failure for these values – this probability is the value of the distribution function. Next, we find the value k as the value of inverse normal distribution function. We use absolute value of k to avoid confusion with the sign. Given the variation coefficient v_w, we find σ_u using (3). Then we calculate $M_u=ln(N_t)+k\sigma_u$ and $Me_w=e^{M_u}$. Given the value $\bar{n}_N$, we determine $[N]=Me_w/\bar{n}_N$ and $n^*_N=N_t/[N]$ – the test residual durability factor (experimentally confirmed).

For a certain values of the number of cycles N, where $0 \le N \le N_t$ (N can act as $[N]$), we can obtain the probability of failure $P_{(-)N}=F(k)=F\left(-\frac{M_u-ln(N)}{\sigma_u}\right)$. Expanding this expression in more detail, we obtain

$$P_{(-)N}=F(k)=F\left(-\left(ln\left(\frac{N_t}{N}\right)-\left(F^{-1}\left(\frac{1}{n_c+1}\right)\right)\times\sqrt{ln\left(v_w^2+1\right)}\right)\Big/\sqrt{ln\left(v_w^2+1\right)}\right).$$

Since we agreed that parameter k is positive, then when calculating the function $F(k)$, we put a minus sign in front of k. If $N=[N]$, then $P_{(-)N}=P_{(-)[N]}$.

By substituting according to formula (3): $\sigma_u = \sqrt{ln\left(\nu_w^2 + 1\right)}$, we obtain dependencies $N_t = N \cdot exp\left(\left(F^{-1}\left(\frac{1}{n_c+1}\right) - F^{-1}\left(P_{(-)N}\right)\right)\sigma_u\right)$, and $n_c = \frac{1}{F\left(\frac{1}{\sigma_u} ln\left(\frac{N_t}{N}\right) + F^{-1}\left(P_{(-)N}\right)\right)} - 1$.

From this setup and the probability of failure at a given number of cycles N (i.e. $[N]$), we can obtain: from the first – the number of test loading cycles N_t, at a certain number of tested cylinders n_c; from the second – the number of cylinders n_c that must be tested with a given number of test cycles N_t.

In Figure 5 shows dependencies between $n_N^* = N_t/[N]$ and n_c for a lognormal distribution ($[N]$ taken as N) and different failure probabilities $P_{(-)[N]}$, with variation coefficients of 0.2 and 0.5.

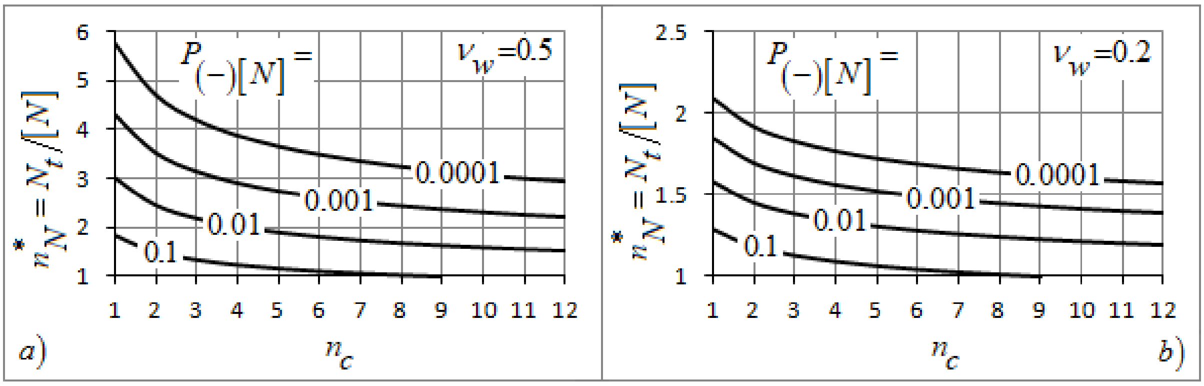

Fig. 5. Relationship between the test durability safety factor for and the number of tested cylinders for different failure probabilities, with variation coefficients of a) 0.5 and b) 0.2.

It should be noted that $n_N^* = N_t/[N]$ can also be less than 1. This occurs in cases where the given probability $P_{(-)[N]}$ is greater than $P_{(-)} = F\left(N_t\right) = 1/\left(n_c + 1\right)$.

In case of a normal distribution, it is easy to show that $M_x = N_t/\left(1 - k\nu_x\right)$, (parameter k here corresponds to N_t), and $[N] = M_x/\overline{n}_N$. For N respectively (parameter k corresponds to N), we obtain $P_{(-)N} = F(k) = F\left(-\frac{M_x - N}{M_x \nu_x}\right) = F\left(\frac{N}{M_x \nu_x} - \frac{1}{\nu_x}\right)$, or $P_{(-)N} = F(k) = F\left(-\frac{1}{\nu_x} - \frac{N}{N_t}\left(\left(F^{-1}\left(\frac{1}{n_c+1}\right)\right) + \frac{1}{\nu_x}\right)\right)$. Worth noting that normal distribution should be used only in cases where $k\nu_x < 1$.

Similarly, for a normal distribution, we can obtain $N_t = N\dfrac{F^{-1}\left(\dfrac{1}{n_c+1}\right)+\dfrac{1}{\nu_x}}{\dfrac{1}{\nu_x}+F^{-1}\left(P_{(-)N}\right)}$, and $n_c = \dfrac{1}{F\left(\dfrac{N_t}{N}\left(\dfrac{1}{\nu_x}+F^{-1}\left(P_{(-)N}\right)\right)-\dfrac{1}{\nu_x}\right)}-1$. Worth noting that when calculating $F^{-1}\left(P_{(-)N}\right)=k$ directly, the value is negative, and its absolute value must be less than $1/\nu_x$. The normal law gives "nice" results only for small values of the variation coefficient.

Basically, another simplistic option is to use a linear dependence for the probability (see segment CB in Figure 6a). In this case we have $P_{(-)N} = \dfrac{N}{N_t\left(n_c+1\right)}$ for a number of cycles N. The probability density function according to the linear law will be a line parallel to the x–axis (see segment (b) in Figure 6b).

Figure 6 shows a hypothetical case where $n_c = 3$ cylinders were tested for $N_t = 600$ cycles, variation coefficient is taken as $\nu_w = \nu_x = 0.5$. The value of the distribution function for 3 cylinders according to formula (4) is $F(600) = 1/(3+1) = 0.25$. Areas under the distribution density functions for normal and lognormal distributions to the left of $N_t = 600$ cycles are equal (see the line (a) in Figure 6b). These areas are also equal to the area of the rectangle bounded by the axes and lines (a) and (b). In this setup, for normal and lognormal laws, only the values of the distribution functions coincide at the value N_t, while the mathematical expectations and standard deviations do not coincide here.

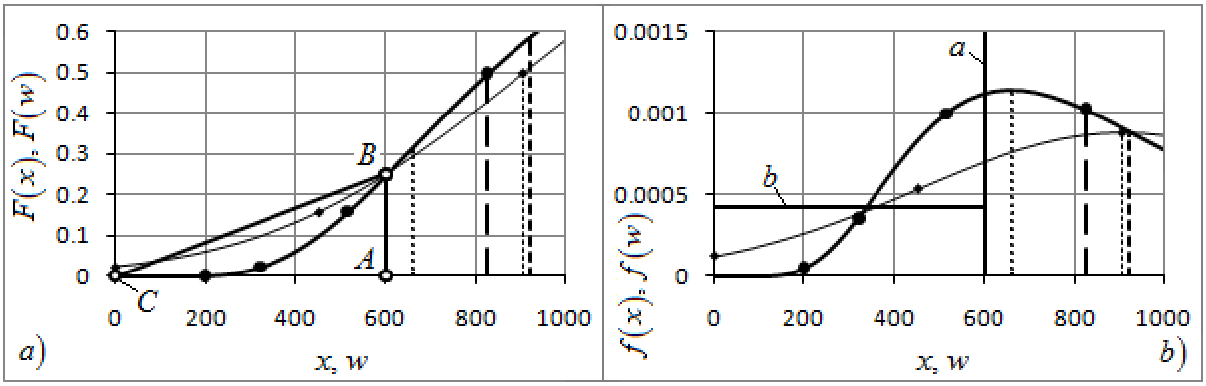

Fig. 6. Distributions when testing of 3 cylinders with 600 loading cycles with a variation coefficient of 0.5: ($N_t = 600, n_c = 3, \nu_w = \nu_x = 0.5$). Notations are similar to Figure 1.

Designing tests for a limited number of cycles, a technical and economic questions may arise. What is more reasonable: to load a larger number of cylinders with a smaller number of cycles or, conversely, a smaller number of cylinders with a larger number of cycles. This question is related to the cost of the cylinders that can no longer be used after testing, and the service life of the testing equipment, especially when it comes to high pressures. An important factor may also be the time that must be spent on cyclic loading and possible equipment repair.

If the number of loading cycles N less than N_t, it is clear that the probability of failure will be less than that determined by formula (4). Figure 7a shows the relation of the probability of failure on the number of loading cycles. For illustration, the dependences are given for cases when 1, 2, 3 and 10 cylinders were subjected to "successful" tests for N_t=1200 cycles. The variation coefficient is taken to be equal to 0.5. The values for N=N_t=1200 correspond to formula (4). The x–scale of these curves, unlike the y–scale, are proportional to each other. For example, multiplying the number of cycles for the curve n_c=3 by the coefficient $872{,}58.../1200$, which corresponds to 0.72715…, it will coincide with the curve for n_c=1. Taking the median, i.e. the value N_t for n_c=1, as 100 %, we find that in case of testing three cylinders, in order to obtain the same probabilities of failure with a small number of cycles, we only need to load them 72.715 % of the cycles. Let's denote this percentage by $(N,\%)$. Figure 7 b shows the failure probabilities w.r.t. the number of loading cycles taken as a percentage of the median. Relations are given for the lognormal and normal distribution laws with variation coefficients of 0.2 and 0.5. The dots indicate the values when testing cylinders 1, 2, 3, 4, 5, 10, 20, 30, 40, 50, 100, (top to bottom). Comparing n_c and $(N,\%)$, for lognormal and normal distributions, with different variation coefficients, we obtain exactly the same relations as in Figure 9, but in order to obtain $(N,\%)$ values on the scale n_N^*, we need to multiply by 10.

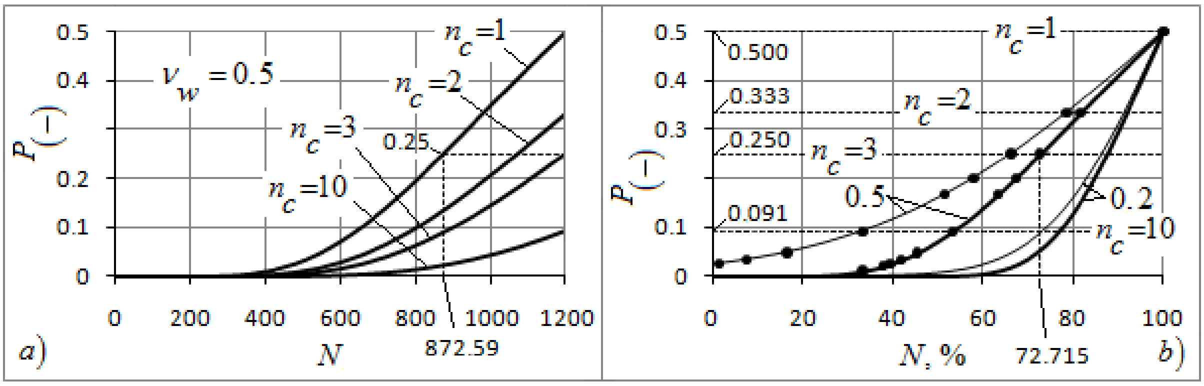

Fig. 7. Probabilities of failure depending on the number of loading cycles:

a – for a lognormal distribution with a variation coefficient equal to 0.5;

b – for lognormal (bold), and normal (thin) distributions. For variation coefficients of 0.5 and 0.2.

Figure 8 shows the probabilities of cylinder failure depending on the number of loading cycles. The distributions are plotted for a "successful" test of three cylinders with 12,000 cycles, as recommended by many regulatory documents.

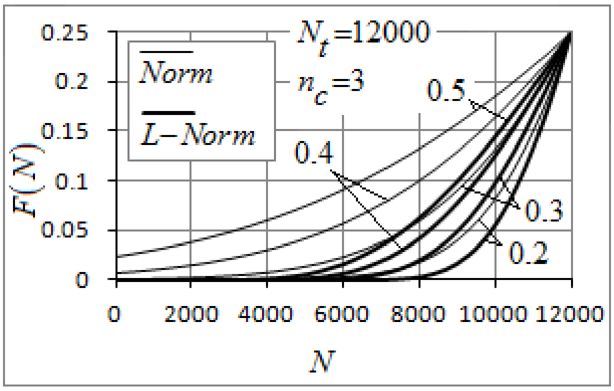

Fig. 8. Probabilities of failure for normal and lognormal distributions depending on the number of loading cycles, for variation coefficients of 0.2, 0.3, 0.4, 0.5, when testing 3 cylinders with 12,000 cycles:

Norm – normal distribution; L–Norm – lognormal distribution.

According to our model, with a "successful" test of one cylinder, we assume a tenfold reserve for durability $n_N^* = \bar{n}_N = 10$, (basically, there may be some other value). With an increase in the number of "successfully" tested cylinders, this coefficient (n_N^*) can certainly be reduced. Should be noted that none of the tested cylinders should "fail" with a number of cycles equal to N_t. Otherwise, other approaches should be used, for example, taking a smaller value for N_t.

The test safety factor for durability for a lognormal distribution, can be rewritten as follows:

$$n_N^* = \frac{N_t}{[N]} = \frac{N_t \bar{n}_N}{Me_w} = \frac{N_t \bar{n}_N}{e^{ln(N_t) + k\sigma_u}} = \frac{\bar{n}_N}{e^{k\sigma_u}} = \frac{\bar{n}_N}{e^{k \cdot \sqrt{ln(v_w^2 + 1)}}}.$$

Relation of the test coefficient of safety for durability (with the standard coefficient equal to 10) to the number of tested cylinders for the variation coefficients 0.2, 03, 04, 0.5 is shown in Figure 9 with bold lines. For example, if 6 cylinders are tested and we use a lognormal distribution, then with a variation coefficient equal to 0.2 it is sufficient to load the cylinders with an eight–fold reserve n_N^*. With a variation coefficient 0.5 it is sufficient to have a six–fold reserve for durability. There is no paradox here, since with a variation coefficient 0.5, the median will be further from the value N_t than with a coefficient of 0.2.

In case of using normal distribution $M_x = N_t + k\sigma_x = N_t + kv_x M_x$. Hence $M_x = N_t / (1 - kv_x)$. Further $[N] = M_x / \bar{n}_N$, and $n_N^* = N_t / [N] = \bar{n}_N (1 - kv_x)$, see Figure 9, thin dotted lines. When using normal distribution law, values of the required reserves will be even lower.

With an increase in the number of tested cylinders (case of "successful" tests), calculated test coefficient of safety for durability will decrease. But this decrease will occur at a slower rate. With an extremely large number of cylinders, it can reach one.

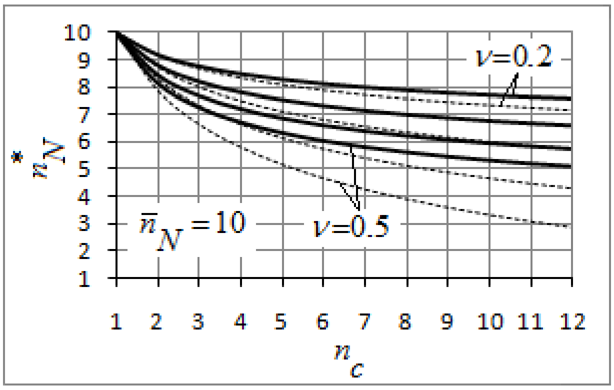

Fig. 9. Test safety factor for durability (with the standard equal to 10), in relation to the number of tested cylinders with variation coefficients of 0.2, 03, 0.4, 0.5: Bold solid lines for lognormal, thin dotted lines for normal distribution.

Obviously, in our case, for the probability that n_c+1 cylinder will not withstand N_t cycles $F(N_t)=P_{(-)}=1/(n_c+1)$, see formula (4), and $k=-F^{-1}(P_{(-)})$. Hence we have $k=-F^{-1}(1/(n_c+1))$. Statement $k=-F^{-1}(P_{(-)})$, where $P_{(-)}=1/(n_c+1)$, is shown in Figure 10a. If we consider $P_{(-)}$, we are to the left of the median, in this case the value k is negative or equal to zero, but for convenience we take it by modulus, implying what is meant. It is clear that for normal and log–normal distributions the number of cycles corresponding to one k do not coincide.

Probability that all n_c results of the cylinder tests will be less than or greater than a given number N_t, according to the probability of destruction $P_{(-)}$ for n_c+1 cylinders, in case of a normal distribution, is shown in Figure 10b. To avoid confusion, keep in mind that $F(N_t)\equiv F(k)\equiv P_{(-)}$.

Figure 10 shows values for testing 1, 2, 3, 4, 5, 10, 20, 30, 40, 50, 100 cylinders (from right to left).

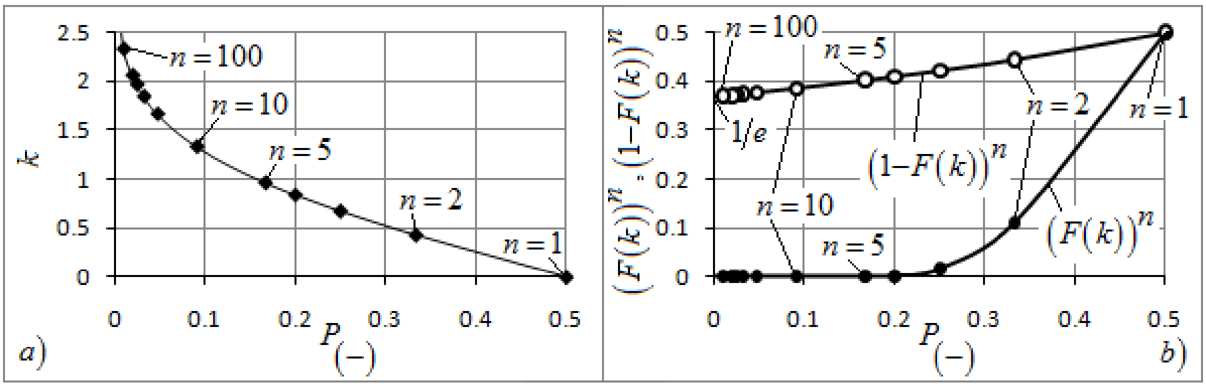

Fig. 10. Dependencies for the case of normal distribution:

Functions: (a) $k=-F^{-1}(P_{(-)})$, where $P_{(-)}=1/(n_c+1)$; (b) $(F(k))^n$ and $(1-F(k))^n$ depending on $P_{(-)}$.

Interesting fact: $\lim_{n_c \to \infty} \left(P_{(+)}\right)^{n_c} = \frac{1}{e} = 0.367879\ldots$, and $\lim_{n_c \to \infty} \left(P_{(-)}\right)^{n_c} = 0$.

It is important to note the following aspect. If, for some reason, we know the median of the durability distribution, then, in case of confidence that durability distribution obeys the lognormal law, standard durability safety factor can be taken as smaller than in case of a normal distribution.

In cyclic testing of objects until failure, we obtain a certain discrete value (N_f), elements of which are loading cycles (n_f) that led to failure of the tested objects. (destruction of one test object is one element n_f of the set N_f). Assuming a normal law of its distribution, we denote it by X ($X \equiv N_f$) with parameters M_x, σ_x, determined by formulas (1). Assuming a lognormal law, we denote it by W ($W \equiv N_f$) with parameters M_w and σ_w also determined by formulas (1). Then, using the values M_w, σ_w we determine M_u and σ_u of the value U, which is normally distributed.

4. Distribution for minimum values, scale factor

Suppose we have some continuous random variable X, a general population, which is described by parameters M_x, σ_x, and has a normal distribution with a distribution function $F(x)$ and probability density function $f(x)$. Consider it as initial distribution. Let us take discrete samples of n elements from this general population. The number of samples of n elements is large enough. From each sample of n elements we take the minimum value. We define the distribution function $F_{min}(x)$ and the probability distribution density $f_{min}(x)$ of the minimum values.

Since the probability that n randomly selected numbers (elements) are simultaneously greater than x, is equal to $(1-F(x))^n$, the distribution function of the minimum values looks like this: $F_{min}(x)=1-(1-F(x))^n$. Due to the fact that $f(x)=\frac{dF(x)}{dx}$, therefore $f_{min}(x)=\frac{dF_{min}(x)}{dx}$, hence $f_{min}(x)=n(1-F(x))^{n-1} \cdot f(x)$, [17, 18].

If $n=1$, then, $F_{min}(x)=F(x)$ and $f_{min}(x)=f(x)$, so the distribution function and probability density function of the minimum values completely coincide with the original distribution functions. The same applies to the numerical parameters: mathematical expectation, mode, median and standard deviation.

It should be noted that for maximum values $F_{max}(x)=(F(x))^n$ and $f_{max}(x)=n(F(x))^{n-1} \cdot f(x)$. The dependencies for maximum values obey the same laws as the dependencies for minimum values.

Functions $f_{min}(x)$ and $f_{max}(x)$, with a symmetric initial distribution, are symmetric to each other w.r.t M_x – mathematical expectation of this initial distribution [17]. Distributions of extreme values themselves are not symmetric, even if the initial distribution is symmetric, see

Figure 12. If the initial distribution is bounded, then distribution of extreme values is also bounded by the same value [17].

Consider the case when the initial distribution obeys the normal law. Let's move from x to k, and take into account only those values that are less than the mathematical expectation. The probability that all of random numbers from n will simultaneously be greater than a certain value determined by k, equals to $P=(1-F(-k))^n$, (see Figure 11b). Therefore $F(-k)=1-P^{1/n}$ and, consequently $k=-F^{-1}\left(1-P^{1/n}\right)$, (see Figure 11a).

For the same k we have $P_1^{1/n_1}=P_2^{1/n_2}$, or $P_2=P_1^{n_2/n_1}$.

Interesting to note that if in the formula $P=(1-F(-k))^n$ values k are taken as a function of $P_{(+)}$, i.e. $k=F^{-1}\left(P_{(+)}\right)=F^{-1}\left(\frac{n}{n+1}\right)$, (see Figure 11c), we have: $P=\left(1-F\left(-F^{-1}\left(P_{(+)}\right)\right)\right)^n=\left(1-F\left(-F^{-1}\left(\frac{n}{n+1}\right)\right)\right)^n$. For $n\Rightarrow\infty$, we have $P\Rightarrow\frac{1}{e}=0.367879\ldots$. For $n=1$, it implies $P=0{,}5$. If the values k are taken as a function of $P_{(-)}$, then $P=\left(1-F\left(F^{-1}\left(P_{(-)}\right)\right)\right)^n=\left(1-\frac{1}{n+1}\right)^n$. Also for $n\Rightarrow\infty$, we have $P\Rightarrow 1/e$.

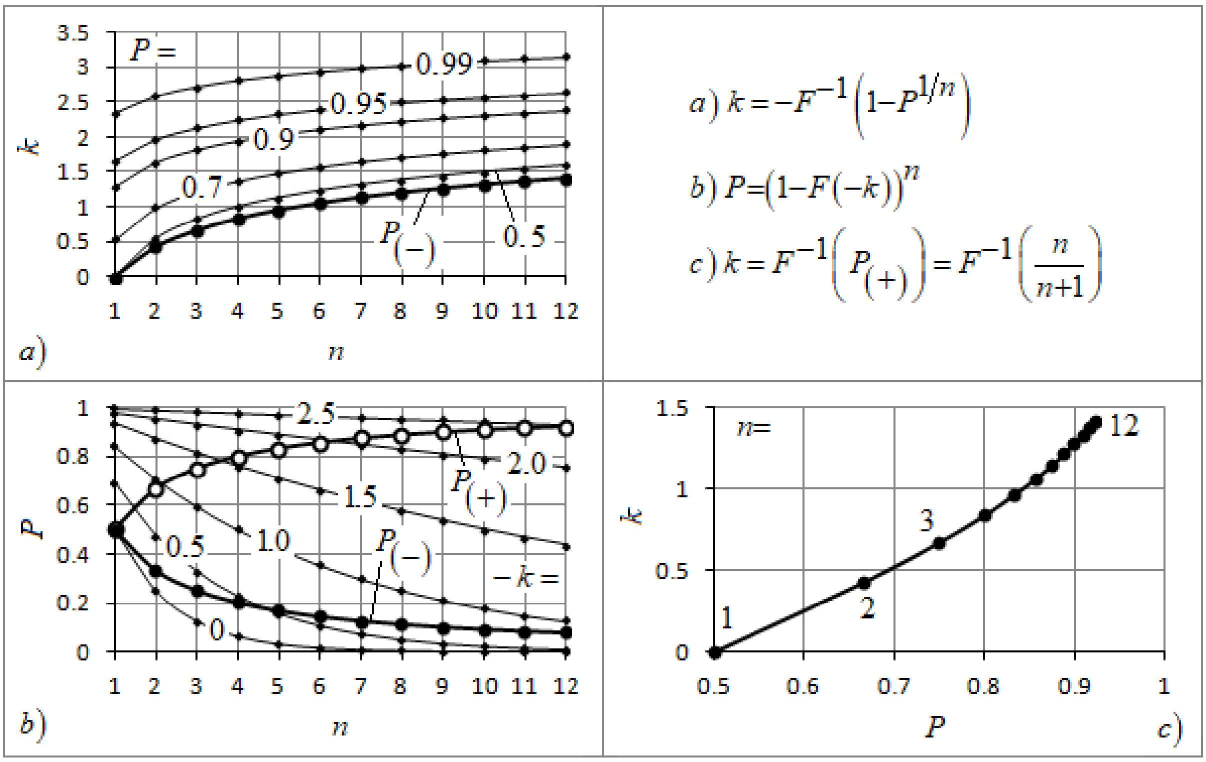

Fig. 11. Normal distribution, for $x\le M_x$: $n=[1, 2, 3, \ldots, 12]$ – *elements.*

In Figure 11 a, with probability P close to 0.4, the calculated values are close to the values when k taken from $P_{(-)}$, see the formulas in Figure 10 a.

It can be shown that, regardless of the initial distribution, the variation coefficient of minimum values is: $v_{min\,x}=\left(\frac{\sigma_{min\,x}}{\sigma_x}\right)\Big/\left(\frac{1}{v_x}-\left(\frac{M_x-M_{min\,x}}{\sigma_x}\right)\right)$, where $\sigma_{min\,x}$ is standard deviation of the minimum values, $M_{min\,x}$ is mathematical expectation of the minimum values.

It is useful to note that median of the minimum values $Me_{min\,x}$, determines the 50 % probability of failure of the structure of n elements as a whole. Let's equate the distribution function of the minimum values to 0.5: $F_{min}(x)=1-(1-F(x))^n=0.5$, therefore $1-F(x)=0.5^{1/n}$ hence $F(x)=1-0.5^{1/n}$. Moving from x to k, we have: $k=-F^{-1}\left(1-0.5^{1/n}\right)$. Hence, median of the distribution of deviations of minimum values from M_x related to σ_x coincides with the value calculated by the formula $k=-F^{-1}\left(1-P^{1/n}\right)$, at $P=0.5$, i.e.: $Me\left\{\frac{M_x-min\,x}{\sigma_x}\right\}=\frac{M_x-Me_{min\,x}}{\sigma_x}=F^{-1}\left(1-0.5^{1/n}\right)$. The median of minimum values $Me_{min\,x}$ does not coincide with the mode $Mo_{min\,x}$ and mathematical expectation $M_{min\,x}$ of minimum values.

When n increases, $\sigma_{min\,x}$ decreases, and $M_{min\,x}$ passes into the region of smaller values, and also $Me_{min\,x}$. $Mo_{min\,x}$. When $n=1$, it implies: $\sigma_{min\,x}=\sigma_x$ and $Me_{min\,x}=Mo_{min\,x}=M_{min\,x}=M_x$.

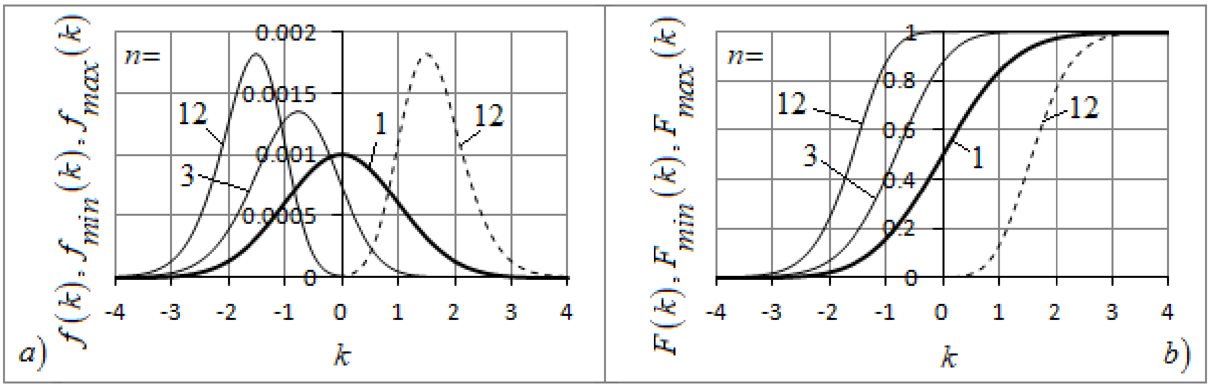

Fig. 12. Distributions of extreme values for the original normal distribution for number of elements 1, 3, 12: Thin solid lines are for minimum values; thin dotted lines are for maximum values. Bold lines are for the main distribution. (for $M_x=800$ and $\sigma_x=400$).

Figure 13 shows the distributions of extreme values for a lognormal initial distribution for the number of elements 1, 3, 12.

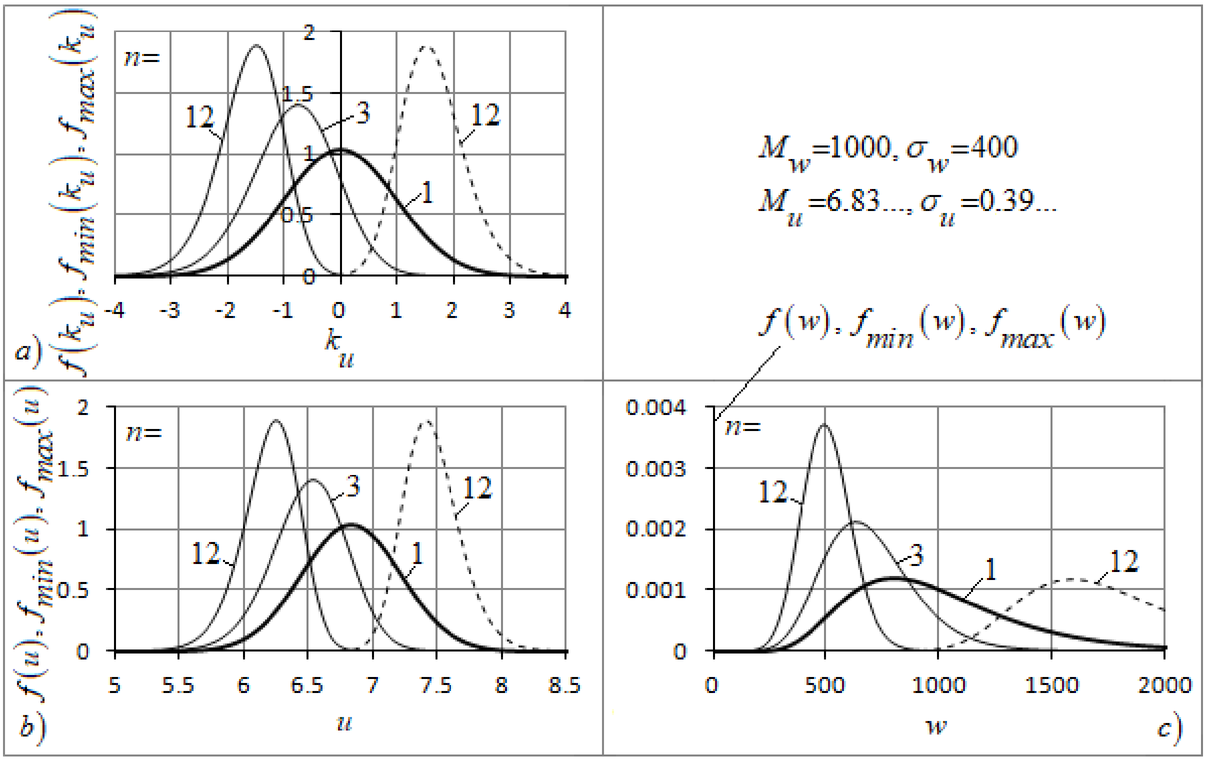

Fig. 13. Distributions of extreme values for lognormal distribution for the number of elements 1, 3, 12: Thin solid lines are for minimum values; thin dotted lines are for maximum values. Bold lines are for the main distribution.

For the distribution of minimum values of the lognormal distribution, using the Monte Carlo method, one can check that $f_{min}(w)=n(1-F(w))^{n-1}\cdot f(w)$, where the argument of the probability density function $f(w)$ is $w=e^u=e^{(M_u \pm k\sigma_u)}$, and the distribution function is $F(w)\equiv F(u)\equiv F(k)$.

$f_{min}(u)=n(1-F(u))^{n-1}\cdot f(u)$, where the argument of the probability density function $f(u)$ is $u=M_u \pm k\sigma_u$, and the distribution function is $F(u)\equiv F(k)$.

Also, $Me_{min\ w}=e^{Me_{min\ u}}$, and $M_{min\ w}=e^{M_{min\ u}+0,5\sigma^2_{min\ u}}$.

With all this, $M_{min\ w}\neq e^{M_{min\ u}}$ mind, $\sigma_{min\ u}\neq\sqrt{ln\left(v^2_{min\ w}+1\right)}$

$Me_{min\ w}\neq M_{min\ w}\Big/\sqrt{v^2_{min\ w}+1}$, And $\sigma^2_{min\ w}\neq e^{2M_{min\ u}+\sigma^2_{min\ u}}\times\left(e^{\sigma^2_{min\ u}}-1\right)$.

Similarly, $f(w)=f(u)\frac{1}{w}$, we have $f_{min}(w)=f_{min}(u)\frac{1}{w}$, where the arguments w and u are related by the dependency $w=e^u$.

Distributions of the logarithm of minimum values of W coincide with the distributions of the minimum values of U. Conversely, distributions of the exponent of minimum values of U, coincide with distributions of the minimum values of W. The same is true between the distributions of the values W and U themselves.

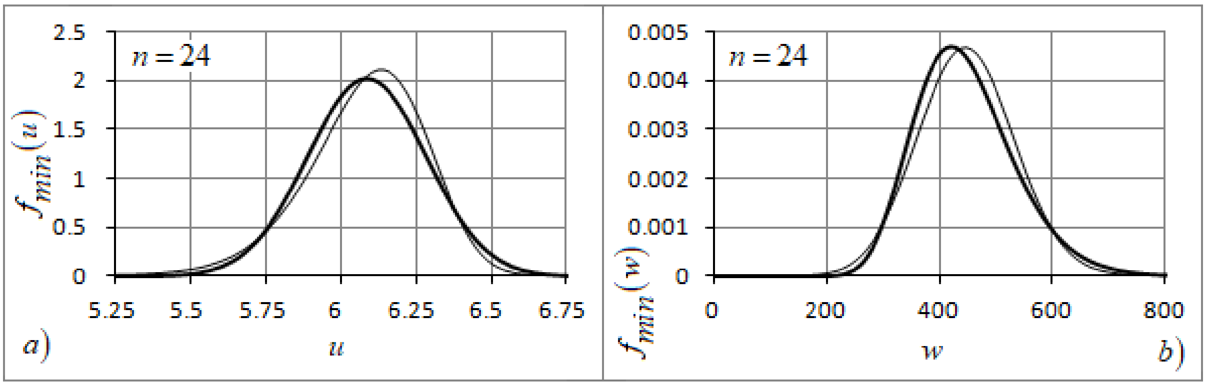

Fig. 14. Distributions of minimum values for 24 elements with conditional M_w=1000 and σ_w=400:

Thin – according to formulas for minimum values. Bold – under the assumption of lognormal distribution.

The number of elements n can act as n–multiplicity. And it is not an invariant value. The distribution of the minimum values of n elements does not coincide with the lognormal law constructed from the minimum values, see Figure 14.

Scale factor. Under conditions of constant stress–strain state (SSS), with an increase in the volume of the material being tested for fatigue life, the mathematical expectation, median and mode will shift to the area of smaller values. The standard deviation will decrease. This can be characterized as the influence of the scale factor.

For example, we subject the same meter–long or five–meter–long pipes to cyclic testing. The pipes must be of small diameter to level out the edge effect from the bottoms. Radiators consisting of radiator sections can also serve as an example. Also, if we increase the diameter and wall thickness of the cylinder without changing the length, but provided that the ratio of diameter to thickness is maintained. The length of welded seams under identical loading conditions can also be considered.

5. Conclusions

1. During cyclic testing of cylinders, according to regulatory documentation, their quasi–static destruction is not permitted.
2. When describing the dispersion of fatigue life (the number of cycles to failure), many authors recommend using a log–normal distribution. In this case, variation coefficient characterizing the dispersion can fluctuate in the range of 0.2...0.5.
3. By using a variation coefficient of 0.5 in the calculations, we predict the worst case scenario.
4. As the number of tested cylinders increases, and if they don't fail, the fatigue life safety factor can be reduced.
5. As the volume of the tested material increases, the fatigue life distribution shifts to lower values, and the standard deviation decreases.

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